TRINITY COLLEGE

Mathematics Specialist
Test 4 2020

Section 1 Calculator Free

Trigonometry

STUDENT'S NAME Solutions

DATE: Wednesday 22nd July **TIME:** 30 minutes **MARKS**: 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Determine the exact value of $\cos 75°$.

$\cos(45+30)$ ✓

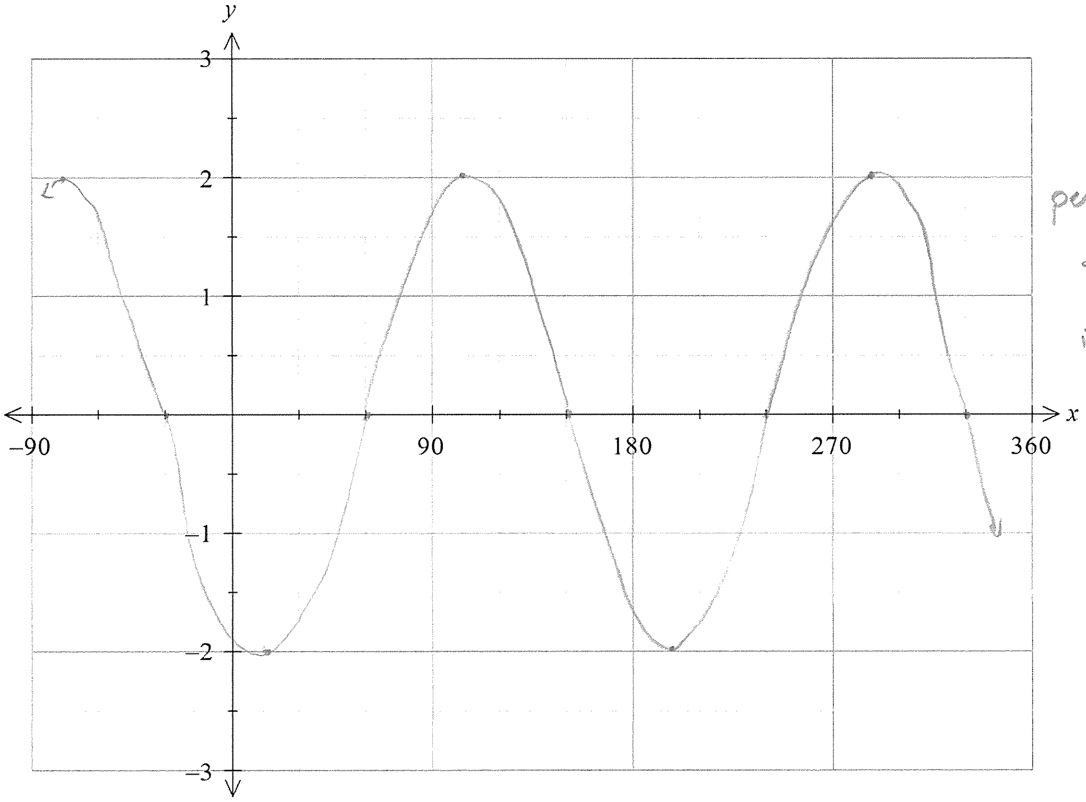
$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ ✓

$= \frac{\sqrt{6} - \sqrt{2}}{4}$ ✓

2. (8 marks)

(a) Sketch the function $y = -2\sin(2(x + 30°))$ on the axes below. [4]



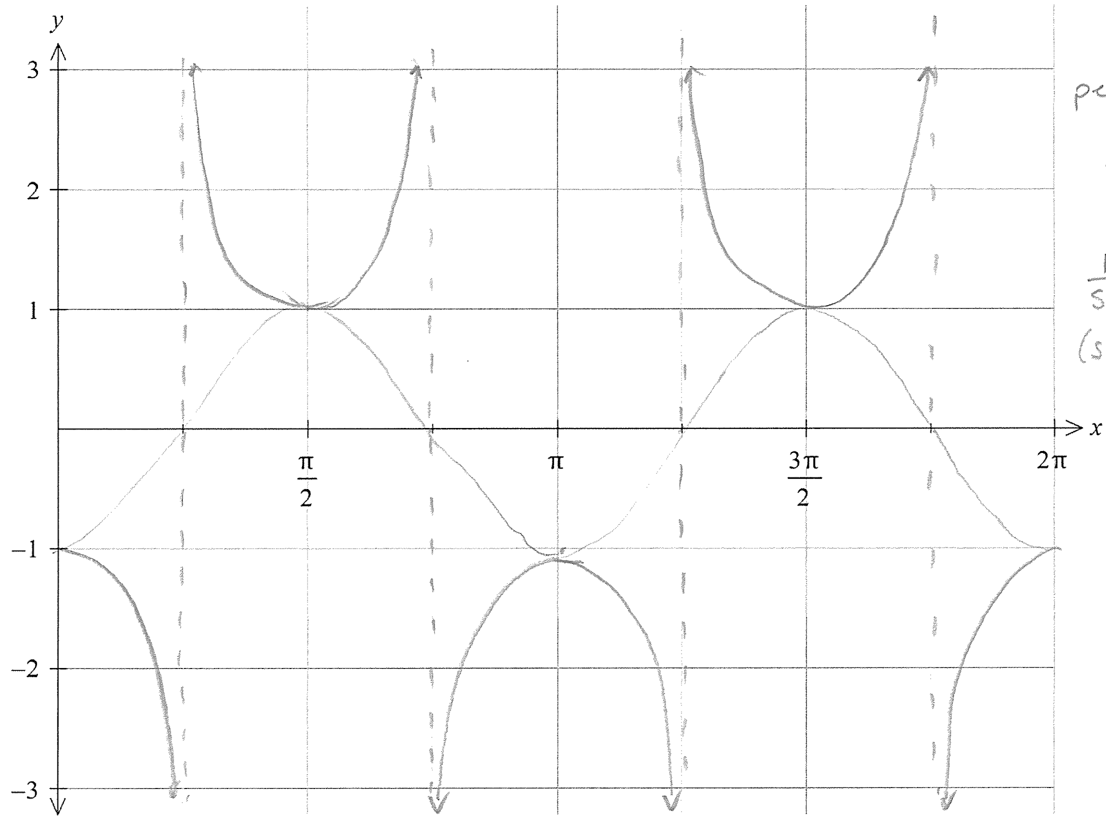
period 180 ✓

← 30° ✓

inverted ✓

↕ × 2 ✓

(b) Sketch the function $y = \operatorname{cosec}\left(2\left(x - \frac{\pi}{4}\right)\right)$ on the axes below. [4]



period = π ✓

→ $\frac{\pi}{4}$ ✓

$\frac{1}{\sin x}$ (shape) ✓

T.P's ✓

3. (8 marks)

Given that $\cos\theta = \dfrac{3}{5}$ where $0 \le \theta \le 90°$, and $\sin\beta = \dfrac{1}{3}$ where $90° \le \beta \le 180°$

Determine:

(a) $\cos(\theta + \beta)$ [4]

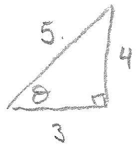
$\cos\theta\cos\beta - \sin\theta\sin\beta$ ✓

$\frac{3}{5} \cdot \frac{-2\sqrt{2}}{3} - \frac{4}{5} \cdot \frac{1}{3}$

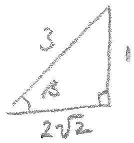
$-\frac{6\sqrt{2}}{15} - \frac{4}{15}$

$\frac{-6\sqrt{2} - 4}{15}$ OR $\frac{-2\sqrt{2}}{5} - \frac{4}{15}$

✓



$\sin\theta = \frac{4}{5}$ ✓



$\cos\beta = \frac{-2\sqrt{2}}{3}$ ✓

(b) $\tan(2\theta)$ [4]

$= \dfrac{2\tan\theta}{1 - \tan^2\theta}$ ✓

$\tan\theta = \frac{4}{3}$ ✓

$= \dfrac{2 \cdot \frac{4}{3}}{1 - \left(\frac{4}{3}\right)^2}$ ✓

$\frac{8}{3} \div \frac{-7}{9}$

$\frac{8}{\cancel{3}_1} \times \frac{\cancel{9}^3}{-7}$

$-\frac{24}{7}$ ✓

4. (8 marks)

Solve

(a) $\tan(2x + 15°) = -1$ for $0° \leq x \leq 180°$ [3]

$\tan(2x+15) = -1$

$0 \leq 2x \leq 360$

$15 \leq 2x+15 \leq 375$

$2x + 15 = 135°, 315°$

$2x = 120°, 300°$

$x = 60°, 150°$

(b) $\sqrt{3}\sin x + \cos x = \sqrt{2}$ for $0 \leq x \leq 2\pi$ by expressing in the form $R\sin(x + \alpha) = c$ [5]

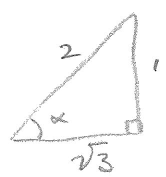
$2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) = \sqrt{2}$

$2(\sin x\cos\alpha + \cos x\sin\alpha) = \sqrt{2}$

$2\sin(x+\alpha) = \sqrt{2}$

$2\sin\left(x+\frac{\pi}{6}\right) = \sqrt{2}$

$\sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$



$\sin\alpha = \frac{1}{2}$

$\cos\alpha = \frac{\sqrt{3}}{2}$

$\therefore \alpha = \frac{\pi}{6}$

$x + \frac{\pi}{6} = \frac{\pi}{4}, \frac{3\pi}{4}$

$x + \frac{2\pi}{12} = \frac{3\pi}{12}, \frac{9\pi}{12}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$

5. (5 marks)

Solve $2\sin^2\theta - \sqrt{3}\sin\theta = 0$ given θ in radians.

$\sin\theta(2\sin\theta - \sqrt{3}) = 0$ ✓

$\sin\theta = 0$ ✓

$\theta = n\pi, \ n \in \mathbb{Z}$ ✓

$2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

✓ $\theta = \frac{\pi}{3} + 2\pi n, \ n \in \mathbb{Z}$

✓ $\theta = \frac{2\pi}{3} + 2\pi n, \ n \in \mathbb{Z}$

or

$(-1)^n \cdot \frac{\pi}{3} + n\pi, \ n \in \mathbb{Z}$

TRINITY COLLEGE

Mathematics Specialist
Test 4 2020

Section 2 Calculator Assumed
Trigonometry

STUDENT'S NAME Solutions

DATE: Wednesday 22nd July **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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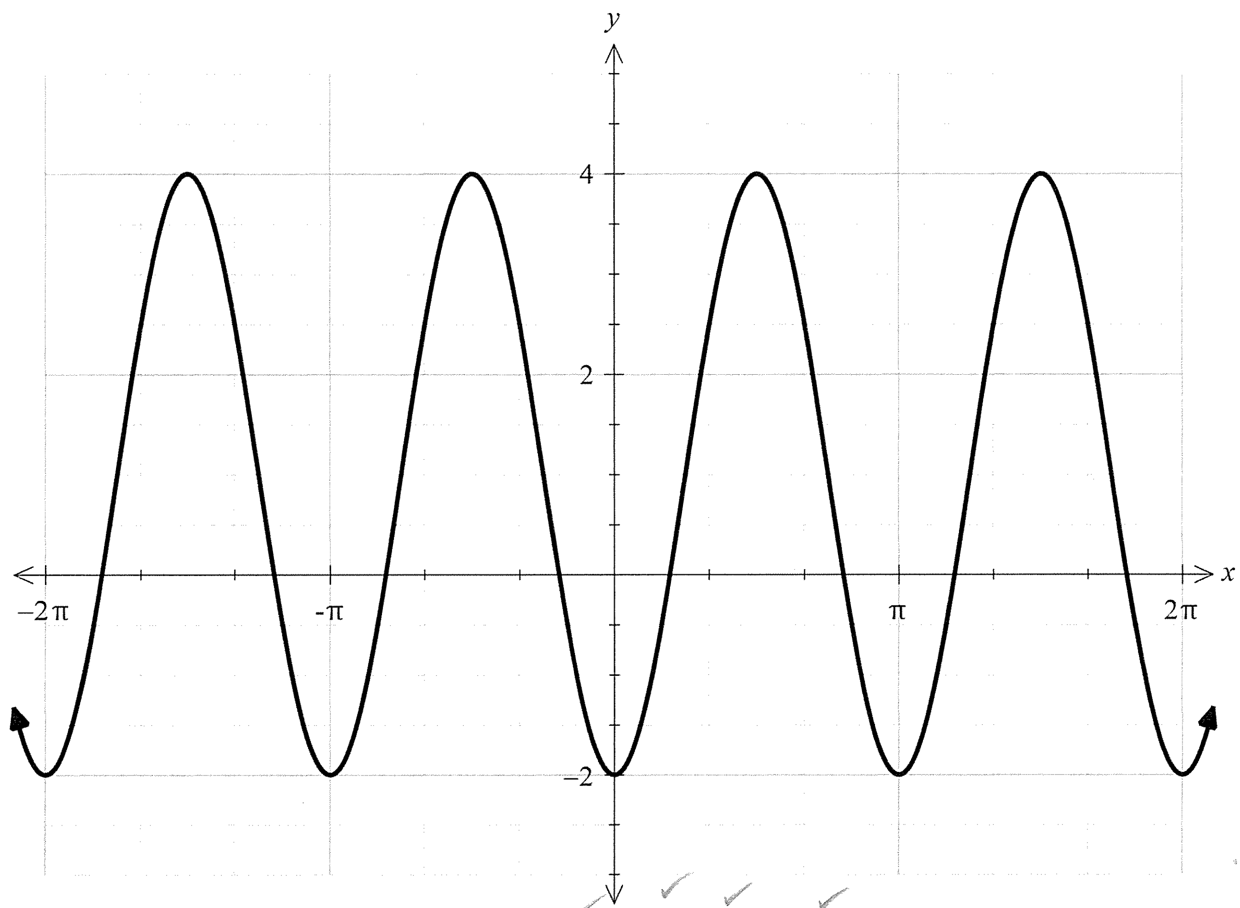
6. (4 marks)

Determine the equation of the function shown below, x in radians



$y = -3\cos(2x) + 1$

7. (9 marks)

A radio wave follows the path of the equation $h = 9\sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$, where h (metres) is the height from a mean level and t (hours) is the time after 9 a.m.

(a) Express $9\sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right)$ in the form $r\sin\left(\frac{\pi t}{4} + \alpha\right)$ [3]

$r^2 = 9^2 + 1^2$

$r = \sqrt{82}$ ✓

$\tan\alpha = \frac{1}{9}$

$\alpha = 0.11157$ ✓

$\sqrt{82}\sin\left(\frac{\pi t}{4} + 0.11157\right)$ ✓

(b) Determine the height of the radio waves at 9 a.m. [1]

1m ✓

(c) Determine the height of the radio waves at 11 a.m. [2]

9m ✓✓

(d) Determine the time(s) in a 24-hour period when the height of the radio waves returns to that of 9 a.m. [3]

heights at 1m ✓

3.72, 8, 11.72, 16, 19.72, 24 ✓

∴ 12:43pm, 5pm, 8:43pm, 1am, 4:43am, 9am.

✓

8. (7 marks)

Prove each of the following.

(a) $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$ [3]

$$
\begin{aligned}
RHS &= \cos(A+B)\cos(A-B) \\
&= (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B) \\
&= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B \\
&= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B \\
&= \cos^2 A - \sin^2 B \\
&= LHS.
\end{aligned}
$$

(b) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\cot\theta+\tan\theta} = \cos\theta$ [4]

$$
\begin{aligned}
LHS &= \frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\left(\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)} \\
&= \frac{\left(1+\frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\left(\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\right)} \\
&= \left(\frac{\sin\theta+\cos\theta}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{1}\right) - \left(\frac{1}{\cos\theta} \div \frac{1}{\sin\theta\cos\theta}\right) \\
&= (\sin\theta+\cos\theta) - \left(\frac{1}{\cancel{\cos\theta}} \times \frac{\sin\theta\cancel{\cos\theta}}{1}\right) \\
&= \sin\theta + \cos\theta - \sin\theta \\
&= \cos\theta \\
&= RHS.
\end{aligned}
$$